Name: _____ Date: _____

Student Exploration: Rotations, Reflections, and Translations

Vocabulary: image, preimage, reflection, rotation, transformation, translation

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
Little Johnnie is playing with a puzzle piece shaped like the letter "b." He is surprised to discover he can make other letters with the same piece.

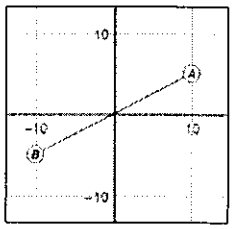
b

1. If he flips the "b" over to the left, what new letter is formed? _____
   Draw a picture to the right.

2. If he flips the letter you drew above down, what new letter is formed? _____
   Draw a picture to the right.

3. Little Johnnie puts the "b" back in its original position. Then he rotates it 90° clockwise. Draw the result to the right. Is this a letter? _____

Gizmo Warm-up
The manipulations of the letter "b" are examples of different **transformations** – **rotations** (turns), **reflections** (flips), and **translations** (slides). In the *Rotations, Reflections, and Translations* Gizmo™, you will rotate, reflect, and translate various figures on a coordinate plane. To begin, select **Segment** from the **Figure type** menu and **Rotate around Origin** from the **Operation** menu.



1. Drag the **Rotation (in degrees)** slider. What happens to $\overline{EF}$? _____

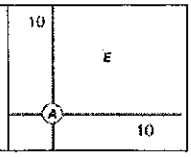
   _____

   In all of the transformations in this Gizmo, $\overline{EF}$ is the **image** and $\overline{AB}$ is the **preimage**.

2. Select **Reflect over *x*-axis**. Drag points *A* and *B* so they are both above the *x*-axis. What do you notice about $\overline{EF}$? _____

3. Select **Translate**. Drag the ***x* translation** and ***y* translation** sliders. What happens to $\overline{EF}$?

   _____

Activity A: **Translations**	Get the Gizmo ready: • Under **Figure type**, select **Point**. • Under **Operation**, select **Translate**.	

1. Recall that point *A* is the preimage, and point *E* is the translated image of point *A*.

   A. With the ***y* translation** slider set to 0, drag the ***x* translation** slider. How does this affect point *E*? _____

   B. Now set the ***x* translation** slider to 0 and drag the ***y* translation** slider. How does this affect point *E*? _____

   C. Set both translation sliders to a positive value. Drag point *A* around. How does this affect point *E*? _____

2. Turn on **Show table**. Set ***x* translation** to –5 and ***y* translation** to 6. Drag point *A* to (–2, 3).

   A. What are the coordinates of point *E*? (_____, _____)

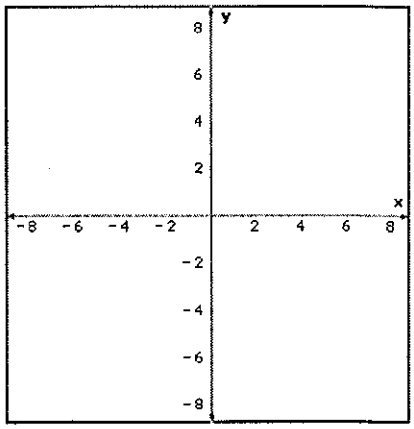
   B. How can you calculate the coordinates of point *E*? _____

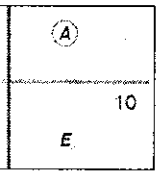
   _____

   C. Suppose a point has coordinates (*x*, *y*). What are the coordinates of the image if the *x* translation is *a* and the *y* translation is *b*? (_____, _____)

3. The endpoints of $\overline{AB}$ are at *A*(–5, 6) and *B*(4, 0). Predict the endpoints of the image $\overline{EF}$ for the translations listed in the table below. Then sketch $\overline{AB}$ and each image on the grid. Click on **Show table** to check your answers.

***x* and *y* translation**	**Point *E*** Image of *A*(–5, 6)	**Point *F*** Image of *B*(4, 0)
x translation: 3 *y* translation: 0		
x translation: –1 *y* translation: –5		
x translation: 1 *y* translation: –6		



Activity B: **Reflections**	Get the Gizmo ready: • Under **Figure type**, select **Point**. • Under **Operation**, select **Reflect over *x*-axis**. • Turn off **Show table**.	

1. Recall that point *E* (the image) is the reflection of point *A* (the preimage).

   A. Drag point *A* up, down, left, and right. Fill in the table to describe how point *E* moves when you do this.

Point *A*	Up	Down	Left	Right
Point *E*				

   B. Turn on **Show table**. Watch the coordinates in the table as you drag point *A* around. How do the coordinates of point *E* compare to the coordinates of point *A*?

_____

_____

   C. A point has coordinates (*x*, *y*). What are the coordinates of the image if (*x*, *y*) is reflected over the *x*-axis? (_____, _____)

2. Turn off **Show table**. Select **Reflect over *y*-axis**.

   A. What do you think will happen to point *E* when point *A* is moved to the right?

_____

   B. Turn on **Show table**. How do the coordinates of point *E* compare to those of point *A*?

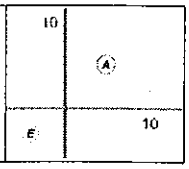
_____

   C. A point has coordinates (*x*, *y*). What are the coordinates of the image if (*x*, *y*) is reflected over the *y*-axis? (_____, _____)

3. Under **Operation**, select **None**. Under **Figure type**, select **Triangle**. Drag the vertices of ΔABC to *A*(7, 5), *B*(–10, 2), and *C*(2, –8). Predict the coordinates of the vertices of the image ΔEFG for the reflections listed below. Then check your answers in the Gizmo.

   Over the *x*-axis: *E*(_____, _____) *F*(_____, _____) *G*(_____, _____)

   Over the *y*-axis: *E*(_____, _____) *F*(_____, _____) *G*(_____, _____)

Activity C: Rotations	Get the Gizmo ready: • Under **Figure** type, select **Point**. • Under **Operation**, select **Rotate around Origin**. • Turn off **Show table**.	

1. Drag point *A*, the preimage, to (10, 5).

   A. Drag the **Rotation (in degrees)** slider. What shape does point *E*, the image of *A*, trace as you drag the slider to the right? _____

   B. Does point *E* move in a clockwise or counterclockwise direction when you drag the slider to the right? _____

   C. Give the angle of rotation that places point *E* at the coordinates listed in the table to the right. Click on **Show table** to check your answers.

   D. What angle of rotation brings point *E* back to point *A*? _____

Point *E* Image of *A*(10, 5)	**Angle of rotation**
(–5, 10)	
(–10, –5)	
(5, –10)	

2. Turn off **Show table**. Select **Segment** under **Figure type**. Set **Rotation (in degrees)** to 0°. Place the endpoints of $\overline{AB}$ at *A*(–1, 10) and *B*(8, –3).

   A. Predict the endpoints of the image $\overline{EF}$ for each angle of rotation listed in the table below. Click on **Show table** to check your answers.

Angle of rotation	**0°**	**90°**	**180°**	**270°**	**360°**
Point *E* Image of *A*(–1, 10)					
Point *F* Image of *B*(8, –3)					

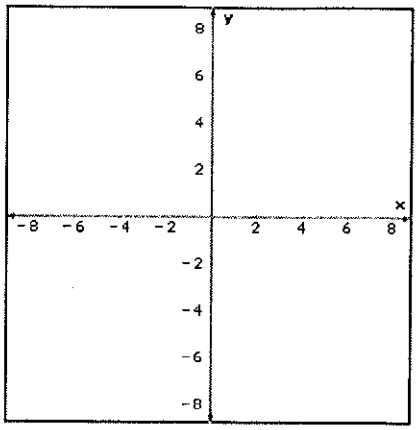
   B. Based on the patterns you have observed, write the general coordinates of the image of a point with coordinates (*x*, *y*) in the table below.

Angle of rotation	**0°**	**90°**	**180°**	**270°**	**360°**
Coordinates of image of (*x*, *y*)					

(Activity C continued on next page)

Activity C (continued from previous page)

3. Suppose ΔABC has the vertices listed below. Write the coordinates of the vertices of image ΔEFG for a 270° rotation in the table. Sketch both triangles on the grid to the right. Then check your answers in the Gizmo.



Vertices of ΔABC	Vertices of ΔEFG
$A(2, 3)$	
$B(-5, 1)$	
$C(4, -6)$	

4. Extension: Select **Point** under **Figure type** and **Rotate around Origin** under **Operation**. Be sure **Show table** is turned on.

   A. You can use the cosine (cos) and sine (sin) of the angle of rotation to find the image of any point on the positive x-axis. To see how this works, first use a calculator to find the cosine and sine (to the nearest hundredth) of the angles in the table below.

Angle of rotation (θ)	cos θ	sin θ	Point E Image of $A(10, 0)$
30°			
45°			
60°			
90°			

   B. Drag point A to (10, 0). Find the coordinates of the image point E for each angle of rotation listed above. Write the coordinates of point E in the last column of the table.

   C. Compare the cosine and sine for each angle to the coordinates of point E. What do you notice? _____

   _____

   Experiment with other points on the positive x-axis and other angles to verify.

   D. The point $(x, 0)$ is rotated through an angle of θ when $x > 0$. Based on the patterns you have observed, use cosine (cos) and sine (sin) to write a formula for the coordinates of the image.

   (_____, _____)